Int J Theor Phys (2009) 48: 1859–1871
DOI 10.1007/s10773-009-9956-6

Interpretation for Probability Wave and Quantum Measured Problem

Z.S. Wang · R.S. Wu

Received: 25 November 2008 / Accepted: 27 January 2009 / Published online: 12 February 2009

Abstract By using Hamilton-Jacobi-Bellman equation with complex time, we investigate quantum theory in timelike curve. State vectors of a physical system in the two-dimensional timelike curve not only obey Schrödinger equation in the observed timespace but also involve random motion in the traversal timespace. The random motion with hidden variables is successfully to explain why the wave function is a probability wave. Quantum measurement are discussed in present work. The results are in agreement with the conventional interpretation of quantum theory.

Keywords Probability wave · Complex time · Schrödinger cat

1 Introduction

Quantum theory is one of the great physical theories of the 20th century. Quantum mechanics has not only profoundly advanced our understanding of nature but has also provided the basis of numerous technologies. Most significantly, quantum mechanics changed our view of the world in a way that was completely surprising and had unprecedented depth. To this date, all experiments magnificently confirm all quantum predictions with impressive precision. However, some fundamental enigmas of quantum theory remain unresolved.

What was the wave function described by the Schrödinger equation and why was the probability wave? This central puzzle of quantum mechanics remains a potential and controversial issue up to now.

Born [1] proposed that the wave function should be interpreted in terms of probabilities. When the location of a microscopic object, such as photon, electron, neutron, proton and atom, was observed by experimenters, the probability of finding it in each region depends on the magnitude of its wave function there. The act of observing the quantum superposi-

Z.S. Wang (✉) · R.S. Wu
College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang 330022, People's Republic of China
e-mail: zishengwang@yahoo.com

tions, however, triggers an abrupt change in its wave function, commonly called a collapse.[1] Measured question for the quantum superposition lies at the heart of quantum mechanics and gives rise to many of its paradoxes.

This interpretation was called as the Copenhagen interpretation, which suggested that a fundamental randomness was built into the laws of nature. This assumption has been the object of severe criticism, notably on the part of Einstein [2], who has always believed that, even at the quantum level, there must exist precisely definable elements or dynamical variables determining the actual behavior of each individual system, and not merely its probable behavior.

The Copenhagen interpretation provided a strikingly successful recipe for doing calculations that accurately described the outcomes of experiments, but the suspicion lingered that some equation ought to describe when and how this collapse occurred.

Instead of being collapsed by measurements, Everett [3] assumed that all measurement results coexist in split universes, where microscopic superpositions would rapidly get amplified into byzantine macroscopic superpositions. In Everett's scenario, that wave function would always evolve in a deterministic way, leaving no room for mysterious nonunitary collapse or God playing dice. Evertt's viewpoint, formally called the relative-state formulation, became popularly known as the many-words interpretation perceives its own word. This viewpoint simplifies the underlying theory by removing the collapse postulate. But the price it pays for this simplicity is in the conclusion that these parallel perceptions of reality are equally real.

The experimental progress [4, 5] of the past few decades was paralleled by great advances in theoretical understanding. Evertt's work had left two crucial questions unanswered. If the word actually contains bizarre macroscopic superpositions, why don't we perceive them?

Bohm tried to replace the seeming quantum randomness by some kind of unknown quantity carried about inside particles-so-called hidden variables, where particles actually have fixed positions and momenta at all times but move in a quantum potentials in terms of a consequence of the Schrödinger equation [6–9]. John S. Bell [10] showed that in this case quantities that could be measured in certain difficult experiments would inevitably disagree with the standard quantum predictions. After many years, technology allowed researchers to conduct the experiments and to eliminate Bohm's hidden variables as a possibility [11, 12], where a photon in two places was observed at once.

At present work, we reinvestigate the Schrödinger equation and collapse of the wave function. The aim is not to deny or contradict the conventional formulation of quantum theory, but wants to understand how a fundamental randomness is built into the laws of nature. By suppling a new, more general and complete formulation, the conventional interpretation of the quantum theory can be deduced.

2 Schrödinger Equation

According to Louis de Broglie [13] matter wave, a physical system is completely described by a state function ψ of the system on an appropriate configuration space in quantum theory. It is known that the wave function of the physical system can be expressed as

$$\psi = R(x,t)\exp(iS(x,t)/\hbar), \tag{1}$$

[1] Essays 1958–1962 on Atomic Physics and Human Knowledge (1963), p. 56.

where $R(x,t)$ is related to the amplitude of the wave function and $S(x,t)$ is a classical action of the system, both being real.

In terms of geometrical optics, there corresponds on the optical side the light ray for the path of the representative point in configuration space. Therefore, the Hamiltonian analogy of mechanics to optics is an analogy to geometric optics. Hamilton's variational principle can be shown to correspond to Fermat's principle for a wave propagation in configuration space, and the Hamilton-Jacobi equation expresses Huygens' principle for this wave propagation. Therefore, Schrödinger thought that a wave equation should be deduced by the Hamilton-Jacobi equation [14–16].

Let's start from Hamilton-Jacobi differential equation,

$$\frac{\partial S}{\partial t}+\frac{1}{2m}(\nabla S)^2+V=0. \tag{2}$$

In the Schrödinger original paper, $R(x,t)$ is regarded as a constant [14–16]. Thus, we have

$$\frac{\partial \psi}{\partial t}=\frac{i}{\hbar}\frac{\partial S}{\partial t}\psi, \qquad \nabla\psi=\frac{i}{\hbar}\nabla S\psi. \tag{3}$$

From (3), one finds

$$\frac{\partial S}{\partial t}=-i\hbar\frac{1}{\psi}\frac{\partial \psi}{\partial t}, \qquad \nabla S=-i\hbar\frac{1}{\psi}\nabla\psi. \tag{4}$$

Inserting (4) into (2), the Hamilton-Jacobi equation becomes

$$\frac{\partial S}{\partial t}+\frac{1}{2m}(\nabla S)(\nabla S)^{+}+V=-i\hbar\frac{1}{\psi}\frac{\partial \psi}{\partial t}+\frac{\hbar^2}{2m}\frac{1}{\psi\psi^{+}}\nabla\psi\nabla\psi^{+}+V=0, \tag{5}$$

where $\nabla S=(\nabla S)^{+}$ is used because S is real.

According to (5), we take a Lagrangian density of the physical system as

$$\mathcal{L}=-i\hbar\psi^{+}\frac{\partial \psi}{\partial t}+\frac{\hbar^2}{2m}\nabla\psi\nabla\psi^{+}+V\psi\psi^{+}, \tag{6}$$

and the solution of the Hamilton variational problem to (6) may be written as

$$i\hbar\frac{\partial \psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V\right]\psi. \tag{7}$$

The Schrödinger equation is readily verified that the $R(x,t)$ and $S(x,t)$ satisfy

$$\frac{\partial R}{\partial t}=-\frac{1}{2m}[R\nabla^2 S+2\nabla R\cdot\nabla S], \tag{8}$$

which describes conservation of probability for the physical system, and

$$\frac{\partial S}{\partial t}=-\left[\frac{(\nabla S)^2}{2m}+V(x)-\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right] \tag{9}$$

which implies that motion of particle is determined not only from classical potential, such as $V(x)$, but also from additional term $\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ called as the quantum potential by Bohm [6, 7]. $R(x,t)$ is determined in terms of the action $S(x,t)$ by the differential equation (8).

If $R(x,t)$ was constant, $\nabla^2 S$ in (8) and the quantum potential in (9) would disappear. However, the Schrödinger equations (7)–(9) with probability interpretation is in excellent agreement with an extremely wide range of experiments. Therefore, there exists a paradox, on one hand, no experimental evidence is known which contradicts the Schrödinger equation, on the other hand, the equation was deduced by a special case. An alternative way of deducing the Schrödinger equation by the path integral [17] has only a complex phase, the action functional.

3 Quantum Theory in Timelike Curve

Supposed that $R = R(x,t)$ is not a constant, (3) becomes

$$\frac{\partial\psi}{\partial t} = \frac{i}{\hbar}\frac{\partial S}{\partial t}\psi + \frac{\partial R}{\partial t}e^{iS/\hbar}, \qquad \nabla\psi = \frac{i}{\hbar}\nabla S\psi + \nabla R e^{iS/\hbar}. \tag{10}$$

Thus, we have

$$\frac{\partial S}{\partial t} = -i\hbar\frac{1}{\psi}\frac{\partial\psi}{\partial t} + i\hbar\frac{1}{R}\frac{\partial R}{\partial t}, \qquad \nabla S = -i\hbar\frac{1}{\psi}\nabla\psi + i\hbar\frac{1}{R}\nabla R. \tag{11}$$

Inserting (11) into (2), one finds

$$\begin{aligned}\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)(\nabla S)^+ + V = -i\hbar\frac{1}{\psi}\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\frac{1}{\psi\psi^+}\nabla\psi\nabla\psi^+ + V \\ + i\hbar\frac{1}{R}\frac{\partial R}{\partial t} - \frac{\hbar^2}{2m}\frac{1}{R^2}(\nabla R)^2 = 0.\end{aligned} \tag{12}$$

Similarly, the Lagrangian density of the physical system is given by

$$\mathcal{L} = -i\hbar\psi^+\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla\psi\nabla\psi^+ + V\psi\psi^+ + i\hbar R\frac{\partial R}{\partial t} - \frac{\hbar^2}{2m}(\nabla R)^2, \tag{13}$$

in terms of (12). If we take both ψ and R as independent variables, the variational solution with the boundaries $\delta\psi^+ = \delta\psi = \delta R = 0$ to (13) is given by

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi, \qquad \nabla^2 R = 0. \tag{14}$$

The Schrödinger equation does, obviously, not satisfy the condition $\nabla^2 R = 0$ for a quantum wave function because of the Hamilton-Jacobi-Bohm equation (9).

In fact, ψ and R are dependent. Equation (12) should be regarded as a equation of motion. Substituting the Schrödinger equation (7) into (12), we have

$$\begin{aligned}\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)(\nabla S)^+ + V &= \frac{1}{\psi\psi^+}\left(i\hbar R\frac{\partial R}{\partial t} + \frac{\hbar^2}{2m}R\nabla^2 R + \frac{i\hbar^2}{2m}R^2\nabla^2 S + \frac{i\hbar}{m}R\nabla R\nabla S\right) \\ &= 0,\end{aligned} \tag{15}$$

which will also give the conservation of probability (8) and condition $\nabla^2 R = 0$, while it is not true of the condition $\nabla^2 R = 0$ for a quantum theory.

If comparing analytical mechanics with geometrical optics, one can show an identity of the principle of least action and Fermat principle. Moreover, the quantization of the Hamiltonian dynamics of mechanical systems leads to the Heisenberg equation [18]. The idea of an imaginary time version of Hamilton-Jacobi equation, called Hamilton-Jacobi-Bellman equation, has been studied for decades. Therefore, it may be a reasonable generalization that the classical traversal time should be included [19–21].

$$\tau = t + i\sigma. \tag{16}$$

Suppose that the Hamilton-Jacobi differential equation for a quantum physical system in the τ-timespace is satisfied,

$$\frac{\partial S}{\partial \tau} + \frac{1}{2m}(\nabla S)^2 + V = 0, \tag{17}$$

which is called Hamilton-Jacobi-Bellman equation. The action integral in the complex timespace can be expressed as

$$W_\tau = \int \rho(x,\tau)\left[\frac{\partial S}{\partial \tau} + \frac{1}{2m}(\nabla S)^2 + V\right] d^3x d\tau, \tag{18}$$

where $\rho(x,\tau)$ is the probability density of the physical system. According to the action integral, Lagrangian density of the physical system is defined as

$$\mathcal{L} = \mathcal{L}\left(\rho, S, \frac{\partial S}{\partial \tau}, \nabla S\right) = \rho\left[\frac{\partial S}{\partial \tau} + \frac{1}{2m}(\nabla S)^2 + V\right], \tag{19}$$

using the Hamilton variational principle with boundaries $\delta\rho(x,\tau) = \delta S(x,\tau) = 0$, one finds

$$\begin{aligned}\delta W_\tau &= \int \left(\frac{\delta \mathcal{L}}{\delta \rho}\delta\rho + \frac{\delta \mathcal{L}}{\delta S}\delta S + \frac{\delta \mathcal{L}}{\delta \frac{\partial S}{\partial \tau}}\delta\frac{\partial S}{\partial \tau} + \frac{\delta \mathcal{L}}{\delta \nabla S}\cdot \delta\nabla S\right) d^3x d\tau \\ &= \int \left(\frac{\partial}{\partial \tau}\left(\frac{\delta \mathcal{L}}{\delta \frac{\partial S}{\partial \tau}}\delta S\right) + \nabla\cdot\left(\frac{\delta \mathcal{L}}{\delta \nabla S}\delta S\right)\right) d^3x d\tau \\ &\quad + \int \left(\frac{\delta \mathcal{L}}{\delta \rho}\delta\rho + \left(\frac{\delta \mathcal{L}}{\delta S} - \frac{\partial}{\partial \tau}\frac{\delta \mathcal{L}}{\delta \frac{\partial S}{\partial \tau}} - \nabla\cdot\frac{\delta \mathcal{L}}{\delta \nabla S}\right)\delta S\right) d^3x d\tau = 0. \end{aligned} \tag{20}$$

The first term in (20) vanishes because of the boundary conditions, the second term gives out the following two equations,

$$\frac{\delta \mathcal{L}}{\delta \rho} = 0, \tag{21}$$

and

$$\frac{\delta \mathcal{L}}{\delta S} - \frac{\partial}{\partial \tau}\frac{\delta \mathcal{L}}{\delta \frac{\partial S}{\partial \tau}} - \nabla\cdot\frac{\delta \mathcal{L}}{\delta \nabla S} = 0. \tag{22}$$

Using (19), we find that (21) will lead to the Hamilton-Jacobi-Bellman equation (17) and (22) will give out the conservation of the probability,

$$\frac{\partial \rho}{\partial \tau} + \frac{1}{m}\nabla\cdot(\rho\nabla S) = 0, \tag{23}$$

where ρ and $\mathbf{j} = \frac{1}{m}(\rho\nabla S)$ are sometimes referred to as the probability and probability current density in the τ-timespace, respectively. The current density $\mathbf{j}$ is the spatial variation of the action of the physical system that determines the strength of the current. The move the action varies with distance, the greater the current.

Note that (23) is also a conservation law expressing the fact that a change in the particle density in a region of space is compensated for a net change in flux into that region in the complex τ-timespace.

It is in general correction in between the real t-timespace and the imaginary σ-timespace. Therefore, there exists the exchange of the current between the two timespace. Thus, matter creation or destruction would tale place in the observed t-timespace. It is impossible for nonrelativistic quantum theory. However, when there doesn't any interaction between the two timespace, (23) will be decomposed into two independent equation according to the real part and the imaginary part. In this case, the conservation of matter is kept in the observed t-timespace.

In order to check if there exists the correction between the two timespace, we consider the action integral in terms of the two-dimensional liketime curve, such as

$$\begin{aligned} W_{(t,\sigma)} &= \int \rho(x,t,\sigma)\left[\frac{\partial S}{\partial \tau} + \frac{1}{2m}(\nabla S)^2 + V\right] d^3x\,dt\,d\sigma \\ &= \int d^3x\,dt\,d\sigma \left[-i\hbar\psi^+\frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m}\nabla\psi\nabla\psi^+ + V\psi\psi^+ \right. \\ &\quad \left. + i\hbar R\frac{\partial R}{\partial t} - iR^2\frac{\partial S}{\partial \sigma} - \frac{\hbar^2}{2m}(\nabla R)^2\right]. \end{aligned} \tag{24}$$

It should be noted if ψ and ψ^+ are regarded as the independent variables, R and S are not the independent invariables. Using the Hamilton variational principle with boundaries $\delta\psi = \delta\psi^+ = 0$, one can obtain the Schrödinger equation,

$$i\hbar\frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi. \tag{25}$$

It is noted that (25) can only describe evaluation of the wave function in the t-timespace. In the following, we try to give some information in the σ-timespace.

Inserting (25) into (17), one finds

$$\frac{\partial S}{\partial \tau} + \frac{1}{2m}(\nabla S)^2 + V = \frac{1}{|\psi|^2}\left(-iR^2\frac{\partial S}{\partial \sigma} - \hbar R\frac{\partial R}{\partial \sigma} + \frac{\hbar^2}{2m}R\nabla^2 R\right) = 0, \tag{26}$$

where the conservation of the probability (23) is used. Thus, the real and imaginary parts of (26) give out the following two equations in the σ-timespace,

$$\frac{\partial S(t,\sigma)}{\partial \sigma} = 0, \tag{27}$$

which means that the state vector of the physical system allows an arbitrary phase factor without relating to σ variable, and

$$\frac{\partial R(t,\sigma)}{\partial \sigma} = \frac{\hbar}{2m}\nabla^2 R(t,\sigma). \tag{28}$$

Equations (27) and (28), from the Hamilton-Jacobi-Bellman equation (17) and the conservation of the probability (23), make $W_{(t,\tau)}$ be the action integral in the two-dimensional timespace.

For t-timespace, we find

$$\frac{\partial S}{\partial t}+\frac{1}{2m}(\nabla S)(\nabla S)^{+}+V=\frac{1}{|\psi|^2}\left(i\hbar R\frac{\partial R}{\partial t}+\frac{i\hbar^2}{2m}R^2\nabla^2 S+\frac{i\hbar}{m}R\nabla R\nabla S\right)+\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}. \tag{29}$$

It is shown in (29) that the imaginary part is only the probability conservation (8), while the real part is the Hamilton-Jacobi-Bohm equation (9). Now, the paradox occur anyway in coordinates of the two-dimensional timelike curves. The wave functions deed satisfy the Schrödinger equation (25) without any confinement in the t-timespace measured by the experimenters. Moreover, they also involve in a random motion described by (27) and (28) in the σ-timespace with hidden variables.

Quantum field theory in the timelike curves has many examples such as the Casimir effect where the expectation value of the energy-momentum tensor fail to obey the weak energy condition [26, 27]. The suggested time asymmetry is useful to explain why in the beginning the universe was so uniform, as evinced by the microwave background radiation left over from the big bang, whereas the end of the universe must be messy.

The imaginary time may be ascribed with the physical notion of the signal velocity of a truncated wave packet [28] and is of importance for dynamic tunneling events. In recent years, the imaginary time is used to study baryon and lepton number violation processes in collision experiments in the TeV range [29–31]. This process is associated with the tunneling between topologically different vacua in the standard electroweak model through the baryon and lepton number anomaly.

4 Interpretation of Probability Wave

Equation (25) and (27)–(28) show that there doesn't exist any interaction of coupling term between the t- and σ-timespace. In the following, therefore, we will denote S_t and S_σ as phases of wave functions in the t- and the σ-timespace respectively. Similarly, R_t and R_σ are amplitudes of wave functions in the t- and the σ-timespace respectively.

Equations (27) and (28) are diffusion equations, the relevant solutions may be expressed as

$$S_\sigma(\xi-x)=\chi(\xi-x), \tag{30}$$

where χ is an arbitrary function, and

$$R_\sigma(\xi-x)=\frac{1}{(4\pi D\sigma)^{3/2}}e^{-\frac{(\xi-x)^2}{4D\sigma}},\quad D=\hbar/2m, \tag{31}$$

where D is a diffusion coefficient. In the Nelson's stochastic theory, the diffusion coefficient $D=\hbar/2m$ was supposed to derive the Schrödinger equation [22, 23].

It is noted that $\lim_{\sigma\to 0}R_\sigma(\xi-x)=\delta(\xi-x)$. Therefore, ξ is a random variable represented by the position $x(t,\sigma)$. The diffusion process, which is strongly irreversible, is related to a random motion of the Brownian particles.

If setting $k_i = (\xi - x)_i/L_i$ and $n_i = 2\sigma D/L_i^2\,(i = 1, 2, 3)$, where L_i is the mean free path along x, y and z axes respectively; and using the following relation,

$$\lim_{x\to 0}(1-x)^{1/x} = e^{-1} = \lim_{x\to 0}(1+x)^{-1/x}, \tag{32}$$

one finds

$$\begin{aligned}
\exp\left(-2\frac{(\xi-x)^2}{4D\sigma}\right) &= \exp\left(-\frac{(\xi-x)^2}{L^2}\frac{L^2}{2D\sigma}\right) = \exp(-k^2/n) \\
&\simeq \left[\left(1+\frac{k}{n}\right)^{-n/k}\right]^{k^2/2n}\left[\left(1-\frac{k}{n}\right)^{n/k}\right]^{k^2/2n} \\
&= \left(1+\frac{k}{n}\right)^{-k/2}\left(1-\frac{k}{n}\right)^{k/2} \\
&= \frac{n^n(\frac{1}{2})^n}{[\frac{1}{2}(n+k)]^{(n+k)/2}[\frac{1}{2}(n-k)]^{(n-k)/2}}.
\end{aligned} \tag{33}$$

Thus,

$$\begin{aligned}
R_\sigma^2(\xi - x)d^3\xi &= \frac{1}{(4\pi D\sigma)^3}e^{-2\frac{(\xi-x)^2}{4D\sigma}}d^3(\xi - x) \\
&= \frac{1}{\sqrt{4\pi D\sigma}}e^{-\frac{(\xi-x)_1^2}{4D\sigma}}d\frac{(\xi-x)_1}{\sqrt{4\pi D\sigma}} \\
&\quad\times \frac{1}{\sqrt{4\pi D\sigma}}e^{-\frac{(\xi-x)_2^2}{4D\sigma}}d\frac{(\xi-x)_2}{\sqrt{4\pi D\sigma}} \\
&\quad\times \frac{1}{\sqrt{4\pi D\sigma}}e^{-\frac{(\xi-x)_3^2}{4D\sigma}}d\frac{(\xi-x)_3}{\sqrt{4\pi D\sigma}} \\
&\simeq \frac{n_1!}{[\frac{(n+k)_1}{2}]![\frac{(n-k)_1}{2}]!}\left(\frac{1}{2}\right)^{n_1}(\Delta k_1/\sqrt{n_1}) \\
&\quad\times \frac{n_2!}{[\frac{(n+k)_2}{2}]![\frac{(n-k)_2}{2}]!}\left(\frac{1}{2}\right)^{n_2}(\Delta k_2/\sqrt{n_2}) \\
&\quad\times \frac{n_3!}{[\frac{(n+k)_3}{2}]![\frac{(n-k)_3}{2}]!}\left(\frac{1}{2}\right)^{n_3}(\Delta k_3/\sqrt{n_3}),
\end{aligned} \tag{34}$$

where the following approximation is used,

$$n! \simeq (2\pi n)^{1/2}\left(\frac{n}{e}\right)^n. \tag{35}$$

Equation (34) is a probability to take $k_i = (\xi - x)_i/L_i\,(i = 1, 2, 3$ represent x, y, z axes respectively) more jumps in the positive direction after $n_i = 2\sigma D/L_i^2$ jumps in 3-dimensional ensemble.

Taking t and σ as independent variables, the state vector of the physical system may be expressed as

$$\begin{aligned}\Psi(\xi,\sigma,x,t) &= R_\sigma(\xi-x)\exp(i\chi(\xi-x)/\hbar)\psi(x,t)\\ &= R_\sigma(\xi-x)R_t(x,t)e^{i(S_t(x,t)+\chi(\xi-x))/\hbar},\end{aligned}\tag{36}$$

where $\psi(x,t)=R_t(x,t)\exp(iS_t(x,t)/\hbar)$ is determined by the Schrödinger equation (25), while $R_\sigma(\xi-x)\exp(i\chi(\xi-x)/\hbar)$ satisfies (27) and (28). Because R_σ^2 is joint probability distributions in terms of (34) explained as the random motion, Ψ is a probability wave with probability in the observed timespace,

$$\begin{aligned}\rho(t,x) &= \frac{\int d^3\xi|\Psi(\xi,\sigma,x,t)|^2}{\int d^3\xi d^3x|\Psi(\xi,\sigma,x,t)|^2}\\ &= \frac{\int d^3\xi|R_\sigma(\xi-x)R_t(x,t)e^{i(S_t(x,t)+\chi(\xi-x))/\hbar}|^2}{\int d^3\xi d^3x|R_\sigma(\xi-x)R_t(x,t)e^{i(S_t(x,t)+\chi(\xi-x))/\hbar}|^2}\\ &= \frac{|R_t(x,t)|^2}{\int d^3x|R_t(x,t)|^2},\end{aligned}\tag{37}$$

which is exactly the same with the conventional probability described by the Schrödinger equation. Equation (37) shows that for observations which may be reduced to position measurements, the two wave functions, Ψ and ψ, give the same predictions. A physical system is completely described by a state function ψ in terms of the t-timespace, which is an element of a Hilbert space, and which furthermore gives informations about probability distributions in terms of the σ-timespace.

The random distribution $R_\sigma^2 d^3\xi$ is independent of the ψ field and dependent only on our degree of information concerning the location of the particle because of $\lim_{\sigma\to 0} R_\sigma(\xi-x)\sim\delta(\xi-x)$, which may be an appropriate interpretation for a causal theory of the quantum mechanics [24, 25].

5 Many Particle System

Similarly, the extension to an arbitrary number of particles is straightforward, and we shall quote only the results here. The Schrödinger equation for N-particle system with the coordinates x_i $(i=1,\ldots,N)$ may be given by

$$i\hbar\frac{\partial}{\partial t}\psi(x_1,x_2,\ldots,x_N,t)=\left[-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\nabla_i^2+V(x_1,x_2,\ldots,x_N,t)\right]\psi(x_1,x_2,\ldots,x_N,t),\tag{38}$$

the wave function also involves additional random motion in the σ-timespace, such as

$$\frac{\partial}{\partial\sigma}R_\sigma(x_1,x_2,\ldots,x_N)=\sum_{i=1}^{N}\frac{\hbar}{2m_i}\nabla_i^2R_\sigma(x_1,x_2,\ldots,x_N),\tag{39}$$

$$\frac{\partial}{\partial\sigma}S_\sigma(x_1,x_2,\ldots,x_N)=0.\tag{40}$$

The above equations are simply a $3N$-dimensional generalization of the similar three-dimensional equations (25) and (27)–(28).

The solution of the differential equations (40) and (41) will in general lead to multiple-valued function, such as

$$R_\sigma(\xi_1,\dots,\xi_N;x_1,\dots,x_N)=\prod_{i=1}^{N}\frac{1}{(4\pi D_i\sigma)^{3/2}}e^{-\frac{(\xi_i-x_i)^2}{4D_i\sigma}}, \tag{41}$$

$$S_\sigma(\xi_1-x_1,\dots,\xi_N-x_N)=\chi(\xi_1-x_1,\dots,\xi_1-x_N), \tag{42}$$

where $D_i=\hbar/2m_i$ is a diffusion coefficient for i-th particle, the correspondent ξ_i is a random variable represented by the position $x_i(t,\sigma)$. Therefore, the wave function described the behavior of every particle in the many particle system,

$$\begin{aligned}\Psi&(\xi_1,\dots,\xi_N,\sigma,x_1;\dots,x_N,t)\\&=R_\sigma(\xi_1,\dots,\xi_N;x_1,\dots,x_N)R_t(x_1,\dots,x_N,t)\\&\quad\times\exp[i(S_t(x_1,\dots,x_N,t)+\chi(\xi_1-x_1,\dots,\xi_N-x_N))/\hbar],\end{aligned} \tag{43}$$

may obviously be explained as the probability wave with probability,

$$\begin{aligned}\rho(t,x_1,\dots,x_N)&=\frac{\prod_{i=1}^N\int d^3\xi_i|\Psi(\xi_1,\dots,\xi_N,\sigma,x_1,\dots,x_N,t)|^2}{\prod_{i=1}^N\int d^3\xi_i\int d^3x_i|\Psi(\xi_1,\dots,\xi_N,\sigma,x_1,\dots,x_N,t)|^2}\\&=\frac{\prod_{i=1}^N\int d^3\xi_i|R_\sigma R_t e^{i(S_t+\chi)/\hbar}|^2}{\prod_{i=1}^N\int d^3\xi_i\int d^3x_i|R_\sigma R_t e^{i(S_t+\chi)/\hbar}|^2}\\&=\frac{|R_t(x_1,\dots,x_N,t)|^2}{\prod_{i=1}^N\int d^3x_i|R_t(x_1,\dots,x_N,t)|^2}.\end{aligned} \tag{44}$$

The wave amplitude, $R_\sigma^2(\xi_1,\dots,\xi_N;x_1,\dots,x_N)R_t^2(x_1,\dots,x_N,t)$, has two interpretations. First, it defines a probability distribution in terms of (44), which explains why a probability wave is in terms of the hidden variables ξ and σ. Secondly, $R_t^2(x_1,\dots,x_N,t)$ described by the Schrödinger equation is equal to the density of representative points $(x_1,\dots,x_N)$ in our $3N$-dimensional ensemble.

6 Quantum Measurement

For any interpretation it is necessary to put the mathematical model of the theory into correspondence with experience. For this purpose, Let's consider the question about live quantum cat. This is one possible outcome of Schrödinger's famous thought experiment, in which a radioactive substance, on emitting a particle, would trigger the release of lethal poison. The problem posed by the experiment is to reconcile the two following facts. The first is that, empirically, cats invariably appear to us either alive or dead. The second is that the conventional Schrödinger equations of motion seem to predict that cats can be in an almost unimaginably bizarre state in which they are neither alive nor dead.

Imagine that we prepare a Schrödinger cat state, the wave functions may be expressed as

$$|cat\rangle=a\frac{|\Psi_1\rangle}{|\langle\Psi_1|\Psi_1\rangle|^{1/2}}+b\frac{|\Psi_2\rangle}{|\langle\Psi_2|\Psi_2\rangle|^{1/2}}, \tag{45}$$

where Ψ_1 and Ψ_2 are two different states in a superposition,

$$\begin{aligned} |\Psi_1\rangle &= R_\sigma(\xi_1 - x)\exp(i\chi_1(\xi - x)/\hbar)\psi_1(x,t) \\ &= R_\sigma(\xi_1 - x)R_{t1}(x,t)\exp[i(S_{t1}(x,t) + \chi_1(\xi - x))/\hbar), \end{aligned} \tag{46}$$

$$\begin{aligned} |\Psi_2\rangle &= R_\sigma(\xi_2 - x)\exp(i\chi_2(\xi - x)/\hbar)\psi_2(x,t) \\ &= R_\sigma(\xi_2 - x)R_{t2}(x,t)\exp[i(S_{t2}(x,t) + \chi_2(\xi - x))/\hbar]. \end{aligned} \tag{47}$$

For a superposition, $\xi_1 \neq \xi_2$, setting $\xi_1 = \xi + c$ and $\xi_2 = \xi - c$, one finds

$$\begin{aligned} \frac{|\langle\Psi_1|\Psi_2\rangle|^2}{|\langle\Psi_1|\Psi_1\rangle||\langle\Psi_2|\Psi_2\rangle|} &= \frac{|\int d^3\xi R_\sigma(\xi_1 - x)R_\sigma(\xi_2 - x)\langle\psi_1|\psi_2\rangle|^2}{\int d^3\xi R_\sigma^2(\xi - x)\int d^3\eta R_\sigma^2(\eta - x)|\langle\psi_1|\psi_1\rangle||\langle\psi_2|\psi_2\rangle|} \\ &= e^{-\frac{c^2}{D\sigma}}\frac{|\langle\psi_1|\psi_2\rangle|^2}{|\langle\psi_1|\psi_1\rangle||\langle\psi_2|\psi_2\rangle|}. \end{aligned} \tag{48}$$

When $C^2/D\sigma \gg 1$, one finds

$$|\langle\Psi_1|\Psi_2\rangle|^2 \sim 0. \tag{49}$$

Thus,

$$\begin{aligned} \langle cat|cat\rangle &= |a|^2 + |b|^2 + e^{-\frac{c^2}{2D\sigma}}a^*b\frac{\langle\psi_1|\psi_2\rangle}{|\langle\psi_1|\psi_1\rangle|^{1/2}|\langle\psi_2|\psi_2\rangle|^{1/2}} \\ &\quad + e^{-\frac{c^2}{2D\sigma}}ab^*\frac{\langle\psi_2|\psi_1\rangle}{|\langle\psi_1|\psi_1\rangle|^{1/2}|\langle\psi_2|\psi_2\rangle|^{1/2}} \\ &\sim |a|^2 + |b|^2. \end{aligned} \tag{50}$$

This way of decoherence offers an explanation for superselection rules. Our solution to the problem is not recourse to any special role for observers and measured apparatus. The discontinuous jump into an eigenstate is only dependent upon the hidden variables, ξ and σ, insight the wave function.

If a system is observed only in states $|\psi_1\rangle$ and $|\psi_2\rangle$, but never in a superposition, then we know that $\xi_1 = \xi_2 = \xi$ in this case. Thus

$$|\Psi\rangle = a\frac{|\Psi_1\rangle}{|\langle\Psi_1|\Psi_1\rangle|^{1/2}} + b\frac{|\Psi_2\rangle}{|\langle\Psi_2|\Psi_2\rangle|^{1/2}}, \tag{51}$$

one finds

$$\begin{aligned} \frac{|\langle\Psi_1|\Psi_2\rangle|^2}{|\langle\Psi_1|\Psi_1\rangle||\langle\Psi_2|\Psi_2\rangle|} &= \frac{|\int d^3\xi R_\sigma(\xi - x)R_\sigma(\xi - x)\langle\psi_1|\psi_2\rangle|^2}{\int d^3\xi R_\sigma^2(\xi - x)\int d^3\eta R_\sigma^2(\eta - x)|\langle\psi_1|\psi_1\rangle||\langle\psi_2|\psi_2\rangle|} \\ &= \frac{|\frac{1}{(4\pi D\sigma)^3}\int d^3\xi e^{-\frac{(\xi - x)^2}{2D\sigma}}|\langle\psi_1|\psi_2\rangle|^2}{\int d^3\xi R_\sigma^2(\xi - x)\int d^3\eta R_\sigma^2(\eta - x)|\langle\psi_1|\psi_1\rangle||\langle\psi_2|\psi_2\rangle|} \\ &= \frac{|\langle\psi_1|\psi_2\rangle|^2}{|\langle\psi_1|\psi_1\rangle||\langle\psi_2|\psi_2\rangle|}. \end{aligned} \tag{52}$$

From (46) and (47), we have

$$\langle\Psi|\Psi\rangle = |a|^2 + |b|^2 + a^*b\frac{|\langle\psi_1|\psi_2\rangle|}{|\langle\psi_1|\psi_1\rangle|^{1/2}|\langle\psi_2|\psi_2\rangle|^{1/2}} + ab^*\frac{|\langle\psi_2|\psi_1\rangle|^2}{|\langle\psi_1|\psi_1\rangle|^{1/2}|\langle\psi_2|\psi_2\rangle|^{1/2}}. \tag{53}$$

In total, our measurement is the square amplitude measure of the wave function determined by the Schrödinger equation in the real t-timespace. The continuous change of state vector of a physical system with the real timespace observed by experimenters is determined by the Schrödinger equation.

The discontinuous jump is brought about by the observation of a quantity with eigenvectors of the Schrödinger equation, such as $|\psi_1\rangle, |\psi_2\rangle, \ldots,$ in which the state vector $|\psi\rangle$ will changed into the state $|\psi\rangle = \sum_i c_i|\psi_i\rangle$ with probability $|\langle\psi_i|\psi\rangle|^2 = c_i^2$.

7 Conclusion

In conclusion, our theory states that a fundamental randomness is introduced into the wave function in terms of the hidden variables. Once the probability distribution is set up in a statistical ensemble of quantum-mechanical systems, then the results predicted for all the measurement processes will precisely be the same in the causal interpretation as in the usual interpretation.

In terms of coordinates of the timelike curves, the wave functions described by the motions of the microscopic objects not only obey the Schrödinger equation in the real t-timespace but also involve additional random motion permitting a detailed causal and continuous description of all processes in the imaginary σ-timespace. This may be reason why the state vector of the physical system is a probability wave. Furthermore, in the complex time coordinate $\tau = t + i\sigma$, the wave functions are solutions of the Hamilton-Jacobi-Bellman equation and the conservation of probability. Thus, it is shown that why microscopic systems seem to posses their familiar classical properties in the experimental measurement.

It is needed to emphasize that our hidden variable theory is different from Bohm's one [6, 7], where the microscopic object have a fixed position and momentum at all time. The experiments seem to reject the interpretation, where a photon in two places at the same time was observed [11, 12]. Our random distribution, $R_\sigma^2 d^3\xi$, is independent of the ψ field described by the Schrödinger equation. Therefore, a microscopic object may exist in different places at the same time. In contrast to the many world interpretation, our interpretation about the quantum measurement is independent of any measured equipments and environmental effects.

In the Nelson's stochastic theory, on the other hand, the Schrödinger equation is deduced by presuming the diffusion coefficient $D = \hbar/2m$. In present work, D is directly from the Hamilton-Jacobi-Bellman equation. It doesn't include, especially, to explain paradoxes of the quantum measurement.

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